Intangible Capital, Tangible Misallocation

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Abstract

The role of intangible capital in production is growing relative to conventional capital. This paper considers the implications of this shift on the misallocation of inputs across public US firms. I show that ignoring intangibles leads to an overestimation of misallocation costs by 54%. The degree of this overestimation gets worse over time, which explains most of the measured deterioration in allocative efficiency in the US in recent years. I find that misallocation is almost twice as severe in sectors that use comparatively more intangibles as in sectors relying more on tangible capital. I calibrate a variable markup model in which the outcome of intangible investments is uncertain and markups increase with firm productivity. I find that it can generate a significant portion of the measured misallocation.

Keywords: misallocation, intangible assets, productivity

JEL classification: E22, O3, O4

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1 Introduction

In the last few decades, developed nations have experienced a huge increase in the importance of intangible assets. The role of machines and factories in production is falling, while the role of information technology, brand value, organizational structure, and technological know-how is rising. The share of intangible assets in total capital input of the US non-farm business sector has doubled between 1948 and 2007 from 17% to 34% (Corrado and Hulten, 2010). A growing literature has been tackling the implications of this shift on the broader economy. In the meantime, numerous papers have documented the large costs of misallocation of capital across firms, focusing on conventional (tangible) capital[1]. Puzzlingly, the measured cost of misallocation of inputs has been rising in the US in recent decades[2].

The goal of this paper is to consider the role of *intangible* capital in generating the observed misallocation of production inputs and in particular its puzzling rise. I argue that ignoring the growing role of intangibles may lead us to overestimate both the extent and the growth of aggregate misallocation. I also point out that misallocation is significantly worse and deteriorating faster in intangible-intensive sectors.

I use firm-level data on publicly traded US companies from Compustat and estimate the intangible investment series for each firm. Intangible investment is comprised of three components: R&D spending, purchases of external intangibles, and a fraction of general and administrative expenses. The stocks of tangible and intangible assets are then constructed from the respective investment series using the perpetual inventory method.

First, I show that the share of intangible assets in total capital has been growing for the last few decades and is now almost equal to the share of conventional tangible assets. Intangible investment, however, significantly exceeds tangible investment (due to the higher depreciation rate of the former). Moreover, I document that a significant portion of the variation in average products

[1] Restuccia and Rogerson (2017) provides a high-level overview.

[2] This has been noted by Kehrig (2015) and Bils et al. (2018) for manufacturing firms.

of tangible capital across firms that is unexplained by a host of observable variables *is* explained by the share of intangibles in a firm's capital. Essentially, when a firm has "too little" conventional capital, measuring its intangibles can reveal that it compensates by employing more of the latter.

To estimate firm-level distortions and the aggregate costs of misallocation I employ the model of Hsieh and Klenow (2009), HK from now on, adding one more factor of production to it: intangible capital. I use estimable expressions provided by the augmented model with intangibles to calculate the extent of misallocation and compare results to the standard HK model without intangibles.

Using the modified HK model to evaluate the costs of inefficient allocation of inputs across firms, I find that misallocation has been getting worse since at least 1987, whether or not we take intangibles into account. However, ignoring the existence of intangibles induces an overestimation of the costs of misallocation by 54% in 2017. Moreover, the overestimation error skyrocketed in the last three decades due to the rising role of intangible capital. This increase in the error accounts for more than half of the observed deterioration in allocative efficiency. When focusing just on the last two decades of the sample, the rising error accounts for virtually the entire increase in misallocation measured using a one-capital model. Omitting intangible assets from misallocation analysis can, therefore, make allocative efficiency and its evolution seem far worse than they actually are. Taking intangibles into account shows that misallocation has increased far less in recent decades and has remained essentially constant since 1998.

To investigate whether the high-intangible sectors or the low-intangible sectors are more distorted, I rank all industries by their intangible/tangible capital ratio in 2017 and split them into two groups at the median. I document that misallocation is almost twice as severe in the intangible-intensive group as in the tangible-intensive one. In addition, it has been growing slightly faster in the former.

To explain the high misallocation in intangible-intensive sectors, I present a partial equilibrium variable markup model based on Edmond et al. (2018). Upon entry, firms choose intangible investment, draw the productivity of their

investment which determines the amount of intangible capital they obtain, and then decide how much to invest into conventional capital. The market structure is monopolistic competition, but the functional form of the sectoral aggregator is such that a firm's optimal markup is increasing in its relative size. This means that the most productive firms choose fewer inputs than they would in the CES case, generating observed misallocation in inputs.

I calibrate the model to the low-intangible sector and find that it can explain a large portion of the observed misallocation. Changing the factor shares to match the high-intangible sector while keeping other parameters at their low-intangible values, I find that the model produces a third of the measured difference in misallocation between the two sectors. It qualitatively generates the negative relationship between firm size and intangible capital share overall as well as a higher average markup and a more dispersed size, tfpr, and marginal product distributions in the high-intangible sector.

Background. The economic literature has faced several challenges presented by the steadily rising role of intangible assets in production. Not only do intangibles make accounting trickier (putting a dollar value on an idea or a brand is less straightforward than measuring the value of a factory building or a truck), but they also behave differently from conventional capital in some important ways (Haskel and Westlake, 2017).

Some papers have worked on addressing the first issue, coming up with measures of intangible investment on sectoral as well as aggregate levels and exploiting them to obtain more accurate estimates of aggregate productivity and growth (**?**McGrattan and Prescott, 2010; McGrattan, 2017). Others have focused on how tangibles and intangibles differ (mainly in the extent they can be used as collateral, with tangible assets being more pledgable[3]) and used these differences to help understand firm dynamics (Chen, 2014; Peters and Taylor, 2017) or to explain recent economic trends like the puzzling simultaneous fall in interest rates and corporate borrowing (Falato et al., 2013; Caggese and Pérez-

[3]Hall and Lerner (2010) explore the reasons why R&D investment is more difficult to finance externally than conventional capital. Some of the reasons apply to other types of intangibles as well.

Orive, 2016; Döttling and Perotti, 2017)[4].

The misallocation literature is no less extensive. Foundational works include Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), who find that distortions causing an inefficient allocation of capital and labor across firms have sizable output costs and account for a significant portion of cross-country differences in productivity. Since then, much of the work has concentrated on figuring out what factors generate the observed distortions. The evidence points to fixed firm-specific factors being important in developing nations[5] and more benign causes like production function misspecification playing a larger role in rich countries (David and Venkateswaran, 2019).

This literature depends on reliable firm-level data on production inputs. And while great strides have been made in adding intangibles to sectoral and aggregate accounts[6], the lack of analogous firm-level estimates has restricted the misallocation literature to consider only the type of capital that is consistently reported on firm balance sheets: tangible fixed assets like buildings, machines, and vehicles. And yet attempting to bring intangibles into this analysis seems worthwhile, considering their increasing role: according to Corrado and Hulten (2010), aggregate investment on intangible assets already exceeds tangible investment in the US. One paper that does this is Caggese and Pérez-Orive (2016), who observe that productivity is more dispersed in intangible-intensive sectors and argue that the shift to non-pledgable assets is interacting with falling interest rates to impede efficient reallocation of capital and to further amplify the secular stagnation. My paper shares this goal of considering the implications of intangibles for the allocation of capital across firms but focuses on characterizing their role in generating observed misallocation[7].

[4]Not all work exploring the implications of the shift toward intangibles focuses on long-term secular trends. Examples of papers considering the role intangibles may play in short-run business cycle dynamics include Pérez-Orive (2016) and Lopez and Olivella (2018)

[5]For example, Bai et al. (2018) argue that a part of capital misallocation in China is due to private firms having harder access to financing than state-owned enterprises.

[6]For example, the Bureau of Economic Analysis now counts expenditures on several types of intellectual property as intangible investment in its input-output tables.

[7]Moreover, their proposed mechanism (collateral constraints) is better at explaining the misallocation of capital *across* sectors rather than within them.

Layout. The rest of the paper proceeds as follows. Section 2 describes the data construction and characterizes the growing role of intangibles. Section 3 defines the modified HK model used to estimate misallocation and lays out the estimation procedure. Section 4 presents the misallocation estimates and explores the composition and dynamics of measured misallocation. Section 5 discusses several potential mechanisms that could be driving some of the empirical findings. Section 6 proposes a variable markup model as one such mechanism and evaluates its performance. Section 7 concludes.

2 Data and Empirical Motivation

I use annual firm-level data on US companies from S&P's Compustat database, which provides balance sheet information for publicly traded firms. I drop firm-year observations with nonpositive capital stock (Compustat variable $PPEGT$), sales ($SALE$), or cost of goods sold ($COGS$). I deflate all dollar quantities with sectoral output deflators from the Bureau of Labor Statistics, except for variables used below to compute tangible investment (deflated with sectoral equipment and structures investment[8] deflators) and intangible investment (deflated with sectoral intellectual property products investment deflator). Using sectoral deflators from the BLS limits me to the 1987–2017 sample. An extension covering 1970–2017 at the cost of using a single deflator for all sectors and series is presented in Section 4.3.

Tangible capital. Compustat firms report both the value of property, plants, and equipment (variable $PPEGT$) and the value of investment into property, plants, and equipment (variable $CAPX$). Chen (2014) argues that the latter is more reliable and less subject to misreporting. I follow her method of constructing the implied tangible capital stock $k_{T,f,t}$ for each firm f in year t from the investment series using the perpetual inventory approach.

First, I define tangible investment $x_{T,f,t}$ as capital expenditure $CAPX$ less

[8]Deflators for investment in equipment and investment in structures are reported separately. I take an average of the two weighted by their shares in total sectoral investment into equipment and structures, also obtained from BLS data.

the sale of physical assets, $SPPE$. Many firms have occasional missing capital expenditure values. In cases when the gap is just one year long, I interpolate for the missing value by averaging the capital expenditures in the adjacent years. Finally, I construct the tangible capital series:

$$k_{T,f,t} = (1 - \delta_{T,f})k_{T,f,t-1} + x_{T,f,t-1} \tag{1}$$

$\delta_{T,f}$ is the implied depreciation rate[9] of firm f. The value of capital stock in the first period the firm is observed is set to the value of $PPEGT$ in that period.

2.1 Intangible Capital Measurement

Firms do not explicitly report the total value of their intangible assets or expenditures on intangible assets. To construct an estimate of these quantities, I use a method based on Peters and Taylor (2017), which is in turn closely related to Falato et al. (2013). My measure of intangible capital is comprised of three components: knowledge capital, organization capital, and externally purchased intangibles.

Knowledge capital. I construct a measure of knowledge capital from annual R&D expense (XRD) using the perpetual inventory method. Individual missing XRD values are interpolated as described above for tangible investment. Any missing values remaining after interpolation are set to 0. The result is investment in knowledge capital, $x_{\text{know},f,t}$. The depreciation rate of knowledge capital δ_{know} is set to 0.15 as in Falato et al. (2013)[10]. Then knowledge capital is defined as

$$\textsc{Knowledge}_{f,t} = (1 - \delta_{\text{know}})\textsc{Knowledge}_{f,t-1} + x_{\text{know},f,t-1} \tag{2}$$

[9]Constructed also as in Chen (2014): $\delta_{T,f} = \mathbb{E}\left[\frac{x_{T,f,t-1}}{\widehat{k}_{T,f,t-1}} - \frac{\widehat{k}_{T,f,t} - \widehat{k}_{T,f,t-1}}{\widehat{k}_{T,f,t}}\right]$, where $\widehat{k}_{T,f,t}$ is the reported capital stock $PPEGT$. I drop observations whose $\delta_{T,f}$ is below 0 or above 1. The median depreciation rate, after all the outliers have been dropped as described below, is 7.5%.

[10]The depreciation rate of 0.15 for capitalized R&D is standard in the literature, although recent estimates of industry-specific depreciation rates suggest that it may be higher than 15% for many industries; for example, see Li and Hall (2018).

Knowledge capital in the first period the firm is observed is set to R&D expenditure in that period divided by the depreciation rate: $\textsc{Knowledge}_{f,1} = \frac{x_{\text{know},f,1}}{\delta_{\text{know}}}$.

Organization capital. Sales, general, and administrative expense ($XSGA$) is used as the basis for organizational investment. I follow Peters and Taylor (2017) in excluding the R&D expenditure (XRD) and in-process R&D expense ($RDIP$) from $XSGA$ where appropriate[11]. I follow Falato et al. (2013) in choosing δ_{org} of 0.2 and Peters and Taylor (2017) in weighing the organizational investment by just 0.3, since much of the spending reported in $XSGA$ is unlikely to be related to developing the organization and instead represents operational costs. The measure of capital is constructed the same as before:

$$\textsc{Organization}_{f,t} = (1 - \delta_{\text{org}})\textsc{Organization}_{f,t-1} + 0.3 \cdot x_{\text{org},f,t-1} \tag{3}$$

External purchases. One type of intangible capital that *is* reported by Compustat firms is externally acquired intangibles, $INTAN$. It includes purchased licenses, patents, trademarks, goodwill, and other similar items. I replace missing values of $INTAN$ with zero. This is already a stock variable and so no assumptions about the depreciation rate are needed. This stock is referred to as $\textsc{External}_{f,t}$ below.

Intangible capital. Finally, the total stock of intangible assets of firm f in year t is defined as the sum of the three components above:

$$k_{I,f,t} = \textsc{Knowledge}_{f,t} + \textsc{Organization}_{f,t} + \textsc{External}_{f,t} \tag{4}$$

In all that follows, I exclude firms from the financial and real estate sectors, firms whose intangible share ($\frac{k_{I,f,t}}{k_{I,f,t}+k_{T,f,t}}$) is less than 0 or greater than 1, and firms whose asset acquisition ratio is greater than 0.2. The final sample accounts for 23% of total private employment in the US in 2017 (as reported by the BLS).

[11] The exact procedure and justification behind it are described in Appendix B.1 of Peters and Taylor (2017).

2.2 Role of Intangibles

In this section, I argue that intangible assets as measured in the previous section constitute an important part of production and that their role has grown in recent decades.

Aggregate Intangibles. To investigate the aggregate behavior of intangibles in my dataset, I compute "aggregate" capital and investment measures for Compustat firms.

Figure 1 shows the evolution of tangible and intangible capital stocks. Intangible capital was just a quarter as large as physical capital in 1987, but is three quarters as large now.

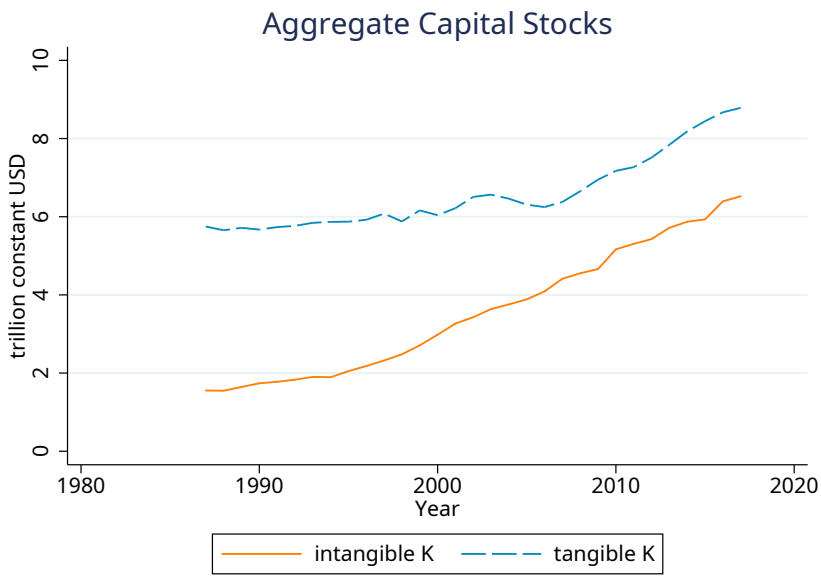


Figure 1: Aggregate tangible and intangible capital

An analogous graph for investment[12] is displayed in Figure 2. Intangible investment has been greater since late 1990s. Intangibles have overtaken tangibles

[12]Intangible investment is the sum of the three component investments. To obtain the investment into externally purchased intangibles, I assume a depreciation rate of 0.15 (same as for R&D).

in investment but not stock because of higher depreciation of intangibles: more investment is needed to maintain the same level of intangible capital.

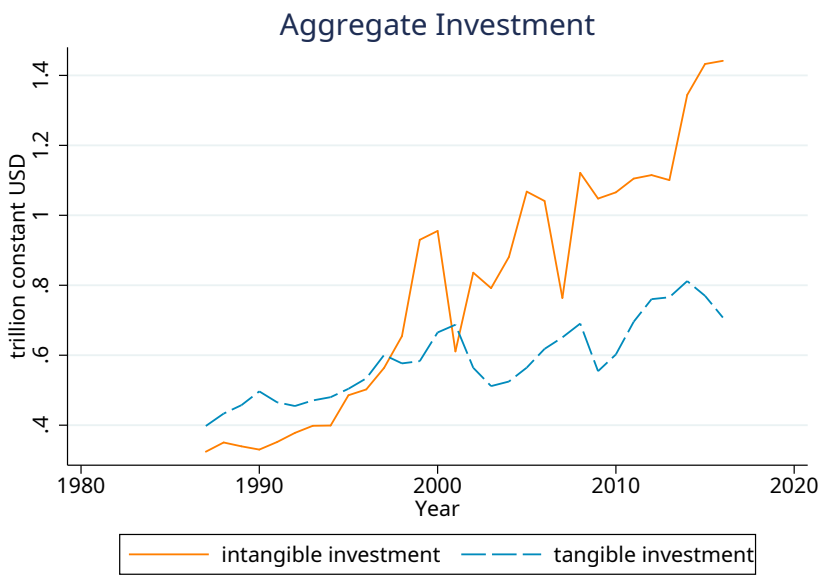


Figure 2: Aggregate tangible and intangible investment

These trends are broadly in line with the aggregate findings of Corrado and Hulten (2010) or Haskel and Westlake (2017), even though the specifics differ due to an imperfect measure of intangibles that I employ and the fact that while Compustat firms account for a sizable portion of the economy, they are far from being entirely representative of the rest.

The evolution of the three components used to construct intangible capital is shown in Figure 3, which displays the share of each component in total intangible capital. The share of external intangibles has increased from a tenth to half over the sample period, most likely due to relabeling.

Intangibles and variation in the average/marginal product of tangible capital. Figure 4 shows how the variation in average revenue products of tangible capital $\text{arpk}_{T,f,t} = \log\left(\frac{p_{f,t}y_{f,t}}{k_{T,f,t}}\right)$ that[13] is unexplained by a host of observables is in part captured by the share of intangibles in total capital: the

[13] $p_{f,t}y_{f,t}$ is sales, using the notation of the model defined below.

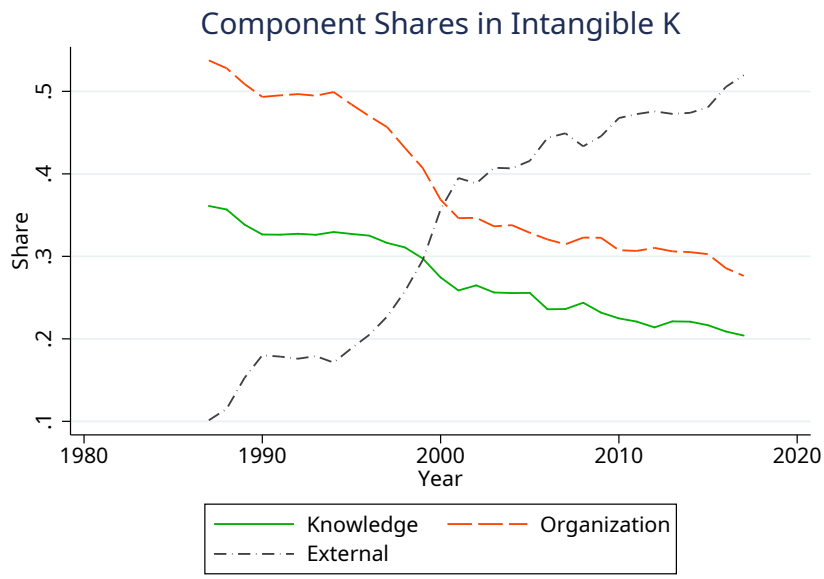


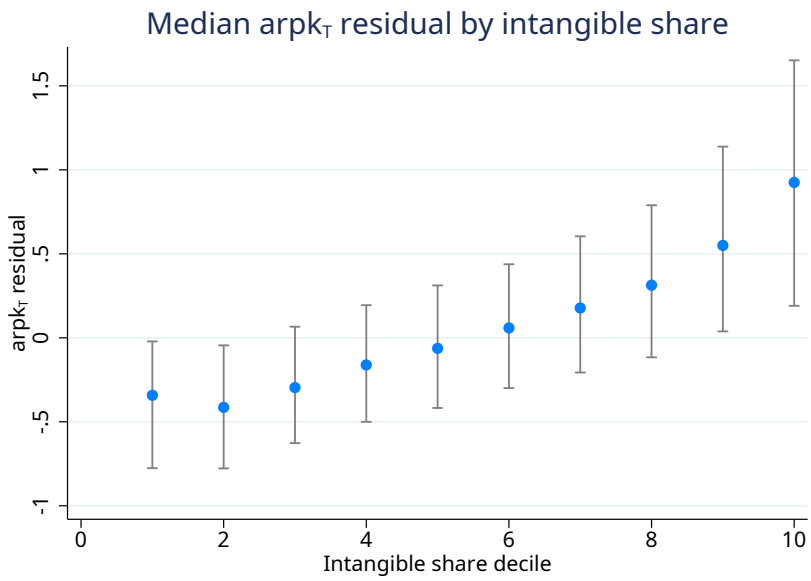
Figure 3: Composition of intangible capital

average revenue product of tangible capital (or, if we assume that the demand elasticity and the capital elasticity are the same within 4-digit NAICS sector-years, the *marginal* revenue product of tangible capital) is higher in firms that use more intangible capital, conditional on sector-year, $\frac{k_T}{l}$ ratio, and other related controls. Firms that use less conventional capital than we would expect from looking at their output and other observables actually employ more intangible assets at the same time.

Intangibles seem to play *some* role in generating the observed heterogeneity in capital products, but how big is this role? To study that, I again regress arpk_T on various observables and look at the marginal increase in R^2 obtained by adding firm intangible share to the list of independent variables:

$$\text{arpk}_{T,f,t} = \beta_0 + \delta[\beta_1 \cdot \textsc{Intan Share}_{f,t}] + \textsc{Controls}_{f,t} + \textsc{Sector-Year FE}_t + \varepsilon_{f,t} \quad (5)$$

where $\delta = 0$ in specifications that don't include the intangible share, and



Note. Residuals obtained from regressing arpk$_T$ on log assets, age, leverage ratio, labor productivity, log employment, k_T/employment ratio, 4 digit NAICS sector-year dummies. Intangible share deciles calculated by sector-year and median of arpk$_T$ taken within those deciles. Bands display the interquartile range.

Figure 4: Unexplained variation in arpk$_T$ and intangible share

$\delta = 1$ in the ones that do.

Results are presented in Table 1. Column (1) includes various controls, but no intangible share, while column (2) adds intangible share to the list of variables. The increase in R^2 attained by this is highlighted. Adding intangible share raises the adjusted R^2 by 0.052 (explanatory power is increased by 10%).

To check whether this explanatory power has changed over time, I repeat the regressions separately on the first and last decades of the sample: 1987–1996 and 2008–2017 respectively. Results are shown in columns (3)-(6) of Table 1. The increase in R^2 in the first decade is lower than in the full sample, 0.050, but in the last decade it is higher, 0.068. The difference in relative gains is even bigger: a 9% increase in the first decade and a 14% increase in the last one.

Table 1: R^2 from regressing arpk_T on various controls w/ and w/o intangible share

	Full sample		1987-1996		2008-2017	
	(1)	(2)	(3)	(4)	(5)	(6)
Intangible share		0.036***		0.052***		0.034***
		(0.000)		(0.001)		(0.001)
Adjusted R^2	0.509	0.560	0.565	0.616	0.492	0.560
Marginal R^2		**0.052**		**0.050**		**0.068**
Controls	Yes	Yes	Yes	Yes	Yes	Yes
NAICS 4 Sector-Year	Yes	Yes	Yes	Yes	Yes	Yes
Observations	112,067	112,067	42,448	42,448	25,369	25,369

Note. Controls include assets, age, leverage ratio, labor productivity, employment, and k_T/employment ratio.
Standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

3 Estimating Misallocation

In the previous section, I have argued that the role of intangible assets in the economy has been growing and that the use of intangible capital is an important dimension of firm heterogeneity. In this section, I modify the model from Hsieh and Klenow (2009), HK below, by adding intangibles to it, and use it to study the role of intangible assets in generating misallocation.

Their model allows me to measure firm-level distortions without taking a stance on the underlying mechanisms that create them. With estimated distortions, I can use the model to quantify the TFP costs of misallocation they generate.

3.1 Model Definition

Overview. My model is identical to HK, except that it adds one more factor of production: intangible capital. It is a static model (to be evaluated in each year independently), and so I omit the time subscripts below. There is a single final good produced by a competitive producer that combines intermediates from S sectors:

$$Y = \prod_{s=1}^{S} Y_s^{\theta_s} \tag{6}$$

where Y_s is the output of sector s and θ_s is its share in the aggregate, with shares summing to 1. The demand of the final good producer for intermediate s is

$$Y_s = P_s^{-1}\theta_s PY \tag{7}$$

where the price level is

$$P = \prod_{s=1}^{S}\left(\frac{P_s}{\theta_s}\right)^{\theta_s} \tag{8}$$

The intermediate good Y_s is itself aggregated by a competitive producer from the varieties $y_{s,f}$ of F_s firms:

$$Y_s = \left(\sum_{f=1}^{F_s} y_{s,f}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{9}$$

The intermediate aggregator's demand for variety $y_{s,f}$ with price $p_{s,f}$ is

$$y_{s,f} = p_{s,f}^{-\sigma} P_s^{\sigma} Y_s \tag{10}$$

where the price level is

$$P_s = \left(\sum_{f=1}^{F_s} p_{s,f}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{11}$$

Firms. Each variety $y_{s,f}$ is produced by a monopolistically competitive firm that uses Cobb-Douglas technology, has productivity $a_{s,f}$, and employs three inputs: tangible capital $k_{T,s,f}$ (paid return r_T), intangible capital $k_{I,s,f}$ (paid return r_I), and variable input[14] $v_{s,f}$ (paid w). The firm is moreover subject to three distortions: output distortion $(1-\tau_{Y,s,f})$, tangible distortion $(1+\tau_{T,s,f})$ and intangible distortion $(1+\tau_{I,s,f})$. The firm's problem is

$$\max_{p_{s,f},v_{s,f},k_{T,s,f},k_{I,s,f}} (1-\tau_{Y,s,f})p_{s,f}y_{s,f} - wv_{s,f} - (1+\tau_{T,s,f})r_T k_{T,s,f} - (1+\tau_{I,s,f})r_I k_{I,s,f} \tag{12}$$

[14]Technically, all three inputs in this static model are variable. I call v the variable input as it is more appropriate from the measurement standpoint, see Section 3.4.

s.t.

$$y_{s,f} = a_{s,f} k_{T,s,f}^{\alpha_{T,s}} k_{I,s,f}^{\alpha_{I,s}} v_{s,f}^{1-\alpha_{T,s}-\alpha_{I,s}}$$

Firm's profit maximization yields the following optimal choices of price and variable input:

$$\begin{aligned} p_{s,f} = & \frac{\sigma}{\sigma-1} \frac{1}{a_{s,f}(1-\tau_{Y,s,f})} \times \\ & \left(\frac{(1+\tau_{T,s,f})r_T}{\alpha_{T,s}}\right)^{\alpha_{T,s}} \left(\frac{(1+\tau_{I,s,f})r_I}{\alpha_{I,s}}\right)^{\alpha_{I,s}} \left(\frac{w}{1-\alpha_{T,s}-\alpha_{I,s}}\right)^{1-\alpha_{T,s}-\alpha_{I,s}} \end{aligned} \tag{13}$$

$$\begin{aligned} v_{s,f} = & \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} P_s^{\sigma} Y_s a_{s,f}^{\sigma-1} (1-\tau_{Y,s,f})^{\sigma} \left(\frac{(1+\tau_{T,s,f})r_T}{\alpha_{T,s}}\right)^{\alpha_{T,s}(1-\sigma)} \times \\ & \left(\frac{(1+\tau_{I,s,f})r_I}{\alpha_{I,s}}\right)^{\alpha_{I,s}(1-\sigma)} \left(\frac{w}{1-\alpha_{T,s}-\alpha_{I,s}}\right)^{\alpha_{T,s}(\sigma-1)+\alpha_{I,s}(\sigma-1)-\sigma} \end{aligned} \tag{14}$$

From here, optimal tangible and intangible capital stocks can be obtained from optimal tangible-variable and tangible-intangible ratios:

$$\frac{k_{T,s,f}}{v_{s,f}} = \frac{w\alpha_{T,s}}{(1-\alpha_{T,s}-\alpha_{I,s})(1+\tau_{T,s,f})r_T} \tag{15}$$

$$\frac{k_{T,s,f}}{k_{I,s,f}} = \frac{\alpha_{T,s}}{\alpha_{I,s}} \frac{(1+\tau_{I,s,f})r_I}{(1+\tau_{T,s,f})r_T} \tag{16}$$

3.2 Deriving Estimable Expressions

By observing a firm's input and output choices and assuming that they represent the optimal solution to the problem in the previous section, we can back out the underlying distortions the firm is facing, as well as its unobserved physical productivity. This is key for quantifying the output costs of misallocation.

Marginal products. We can express marginal revenue products in terms of distortions and input prices:

$$\text{mrpv}_{s,f} = \frac{\partial y_{s,f} p_{s,f}}{\partial v_{s,f}} = \frac{\sigma-1}{\sigma}(1-\alpha_{T,s}-\alpha_{I,s})\frac{y_{s,f}p_{s,f}}{v_{s,f}} = \frac{w}{1-\tau_{Y,s,f}} \tag{17}$$

$$\text{mrpk}_{T,s,f} = \frac{\partial y_{s,f} p_{s,f}}{\partial k_{T,s,f}} = \frac{\sigma - 1}{\sigma} \alpha_{T,s} \frac{y_{s,f} p_{s,f}}{k_{T,s,f}} = \frac{(1 + \tau_{T,s,f}) r_T}{1 - \tau_{Y,s,f}} \tag{18}$$

$$\text{mrpk}_{I,s,f} = \frac{\partial y_{s,f} p_{s,f}}{\partial k_{I,s,f}} = \frac{\sigma - 1}{\sigma} \alpha_{I,s} \frac{y_{s,f} p_{s,f}}{k_{I,s,f}} = \frac{(1 + \tau_{I,s,f}) r_I}{1 - \tau_{Y,s,f}} \tag{19}$$

Importantly, combining these produces expressions for obtaining distortions from elasticities, revenue, and inputs:

$$1 - \tau_{Y,s,f} = \frac{\sigma}{\sigma - 1} \frac{w v_{s,f}}{(1 - \alpha_{T,s} - \alpha_{I,s}) y_{s,f} p_{s,f}} \tag{20}$$

$$1 + \tau_{T,s,f} = \frac{\alpha_{T,s}}{1 - \alpha_{T,s} - \alpha_{I,s}} \frac{w v_{s,f}}{r_T k_{T,s,f}} \tag{21}$$

$$1 + \tau_{I,s,f} = \frac{\alpha_{I,s}}{1 - \alpha_{T,s} - \alpha_{I,s}} \frac{w v_{s,f}}{r_I k_{I,s,f}} \tag{22}$$

Analogously to HK, define sectoral marginal product expressions that can be interpreted as weighted averages of firm marginal products:

$$\text{MRPV}_s = \frac{1}{\sum_{f=1}^{F_s} \frac{y_{s,f} p_{s,f}}{Y_s P_s} \frac{1}{\text{mrpv}_{s,f}}} \tag{23}$$

$$\text{MRPK}_{T,s} = \frac{1}{\sum_{f=1}^{F_s} \frac{y_{s,f} p_{s,f}}{Y_s P_s} \frac{1}{\text{mrpk}_{T,s,f}}} \tag{24}$$

$$\text{MRPK}_{I,s} = \frac{1}{\sum_{f=1}^{F_s} \frac{y_{s,f} p_{s,f}}{Y_s P_s} \frac{1}{\text{mrpk}_{I,s,f}}} \tag{25}$$

Productivity. We can also express firm f's revenue productivity in terms of distortions:

$$\begin{aligned} \text{tfpr}_{s,f} &= p_{s,f} a_{s,f} \\ &= \frac{\sigma}{\sigma - 1} \left(\frac{(1 + \tau_{T,s,f}) r_T}{(1 - \tau_{Y,s,f}) \alpha_{T,s}} \right)^{\alpha_{T,s}} \left(\frac{(1 + \tau_{I,s,f}) r_I}{(1 - \tau_{Y,s,f}) \alpha_{I,s}} \right)^{\alpha_{I,s}} \times \\ &\left(\frac{w}{(1 - \tau_{Y,s,f})(1 - \alpha_{T,s} - \alpha_{I,s})} \right)^{1 - \alpha_{T,s} - \alpha_{I,s}} \end{aligned} \tag{26}$$

Using the demand for firm f's variety, can further obtain an estimable ex-

pression of actual firm TFP:

$$a_{s,f} = \left(P_s Y_s^{\frac{1}{\sigma}}\right)^{\frac{\sigma}{1-\sigma}} \frac{(p_{s,f} y_{s,f})^{\frac{\sigma}{\sigma-1}}}{k_{T,s,f}^{\alpha_{T,s}} k_{I,s,f}^{\alpha_{I,s}} v_{s,f}^{1-\alpha_{T,s}-\alpha_{I,s}}} \tag{27}$$

Moreover, define the sectoral TFPR:

$$\text{TFPR}_s = \frac{\sigma}{\sigma-1} \left(\frac{\text{MRPK}_{T,s}}{\alpha_{T,s}}\right)^{\alpha_{T,s}} \left(\frac{\text{MRPK}_{I,s}}{\alpha_{I,s}}\right)^{\alpha_{I,s}} \left(\frac{\text{MRPV}_s}{1-\alpha_{T,s}-\alpha_{I,s}}\right)^{1-\alpha_{T,s}-\alpha_{I,s}} \tag{28}$$

Then sectoral TFP can be expressed as

$$\text{TFP}_s = \left(\sum_{f=1}^{F_s} \left(a_{s,f} \frac{\text{TFPR}_s}{\text{tfpr}_{s,f}}\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \tag{29}$$

3.3 Reallocation Gains

Equations (20), (21), (22), (27), and (29) (direct counterparts of equations (15), (17), (18), (19) in HK) are key to quantifying misallocation: they provide estimable expressions that allow us to evaluate the output cost of misallocation.

First of all, define efficient sectoral TFP that is attained when all firm distortions within a sector are equal, and thus there is no misallocation of resources:

$$\text{TFP}_{s,\text{eff}} = \left(\sum_{f=1}^{F_s} a_{s,f}^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \tag{30}$$

Reallocation gain, the ratio of efficient output and distorted output, is the main quantity of interest. At the sectoral level, it is

$$\text{Gain}_s = \frac{\text{TFP}_{s,\text{eff}}}{\text{TFP}_s} \tag{31}$$

At the aggregate level, the output gain from equalizing distortions is

$$\text{Gain} = \prod_{s=1}^{S} (\text{Gain}_s)^{\theta_s} \tag{32}$$

3.4 Measurement

Firm variables. Revenue $p_{s,f}y_{s,f}$ is measured as $SALE$ in Compustat. Note that while knowing $P_s Y_s^{\frac{1}{\sigma}}$ is required to calculate $a_{s,f}$ in equation (27), this term cancels out when computing reallocation gains using equation (31), and so it is not needed for quantifying misallocation.

Capital stocks $k_{T,s,f}$ and $k_{I,s,f}$ are measured as described in Section 2. The choice of rental rates r_T and r_I is not directly relevant for the computation of reallocation gains (it only matters for determining factor shares, described below). Because Compustat firms do not report value added or materials expenditure, and very few report labor compensation, I measure $v_{s,f}$ as cost of goods sold ($COGS$ in Compustat), which can be interpreted as the total variable input[15], including not just the costs of labor, but also of materials, energy, and other items.

Factor shares. I cannot separately identify factor shares and distortions, so to compute factor shares, I assume that distortions cancel out once aggregated to the sectoral level. In effect, this means that there is no misallocation across sectors—only across firms within a sector. I measure $1 - \alpha_{T,s} - \alpha_{I,s}$ as the share of sectoral $COGS$ in sectoral $SALE$. I pin down the ratio of the two capital shares, $\frac{\alpha_{T,s}}{\alpha_{I,s}}$, from the ratio of payments to each capital, $\frac{K_{T,s}r_T}{K_{I,s}r_I}$, in each sector. To do this, I assume that the rental rate for each capital is 5% plus depreciation[16], which produces $\frac{r_T}{r_I} = \frac{0.12}{0.22}$. Note that just as all other variables, factor shares are computed for each year independently.

Another way to determine the factor shares is to use the data on factor compensation from the Bureau of Economic Analysis, which reports compensation not only of physical capital and variable inputs, but also of various types of in-

[15] In contrast, Hsieh and Klenow (2009) measure the labor input with payroll and py with value added, which makes my misallocation estimates below not directly comparable to theirs (issues of different firm samples aside).

[16] Tangible depreciation is estimated by applying the implied depreciation procedure from Section 2 to aggregate data and averaging across years, while the intangible depreciation is taken to be the weighted average of the assumed depreciation rates of the three components of aggregate k_I, also averaged across years. Aggregate misallocation estimates are not sensitive to the choice of r_T/r_I.

tangible capital for each sector. Sectoral and aggregate misallocation estimates change little when using this method, and so I stick to the method described in the preceding paragraph, expecting it to be more relevant for my data.

HK without intangibles. In Section 4 below, some results involve comparing the predictions of the standard HK model without intangibles to the modification with intangibles (I call it HK+I) described above. From now on, when I say "HK model", I mean a model with no intangible capital in production, in which the only capital input is set to what I call the tangible capital stock, $k_{s,f} = k_{T,s,f}$. There remains the question of factor shares. Hsieh and Klenow (2009) measure the labor share, $1 - \alpha_s$, as labor compensation in value added. Then they set the capital share α_s to 1 minus the labor share. This means that while the capital *stock* in HK is equivalent to the tangible capital in HK+I, the capital *factor share* in HK is more appropriately mapped to the sum of tangible and intangible factor shares in HK+I: $\alpha_s = \alpha_{T,s} + \alpha_{I,s}$. This assumption on mapping the capital shares of the two models is key in generating the differential predictions of HK and HK+I discussed in Section 4.

What determines the difference in measured misallocation between HK and HK+I? This question can be addressed before going to the data for concrete figures. Let hats denote variables measured by HK. Firm productivity measured by HK can be expressed in terms of HK+I variables:

$$\hat{a}_{s,f} = a_{s,f} \left(\frac{k_{I,s,f}}{k_{T,s,f}} \right)^{\alpha_{I,s}} = a_{s,f} \left(\frac{\alpha_{I,s}(1 + \tau_{T,s,f}) r_T}{\alpha_{T,s}(1 + \tau_{I,s,f}) r_I} \right)^{\alpha_{I,s}} \tag{33}$$

Because the HK model ignores k_I as an input, it overestimates firm productivity by $k_I^{\alpha_{I,s}}$. But because it exaggerates the tangible share by $\alpha_{I,s}$, it simultaneously underestimates firm productivity by $k_T^{\alpha_{I,s}}$. Ignoring intangibles leads us to overestimate a firm's productivity if it's subject to relatively high tangible distortions (since they make it use more intangibles, which are not observed by HK) and to underestimate the productivity if intangible distortions are more severe.

Similarly, the difference between sectoral reallocation gain measured by HK

and the one measured by HK+I can also be expressed[17] purely in terms of HK+I variables:

$$\frac{\widehat{\text{GAIN}}_s}{\text{GAIN}_s} = \frac{\widehat{\text{TFP}}_{s,\text{eff}}}{\widehat{\text{TFP}}_s} \cdot \frac{\text{TFP}_s}{\text{TFP}_{s,\text{eff}}} = \left(\frac{\sum_i \left(a_{s,f} \left(\frac{1+\tau_{T,s,f}}{1+\tau_{I,s,f}} \right)^{\alpha_{I,s}} \right)^{\sigma-1}}{\sum_i a_{s,f}^{\sigma-1}} \right)^{\frac{1}{\sigma-1}} \tag{34}$$

The discrepancy between the two models is driven by HK's misestimation of productivity relative to HK+I. To further interpret this expression, assume that $a_{s,f}, \tau_{T,s,f}, \tau_{I,s,f}$ are jointly lognormally distributed with their logs having means $\mu_{a,s}, \mu_{T,s}, \mu_{I,s}$ and (co)variances $\sigma^2_{T,s}, \sigma_{T,I,s}$, etc. Then

$$\log\left(\widehat{\text{GAIN}}_s/\text{GAIN}_s\right) = \\ \alpha_{I,s}\left((\mu_{T,s} - \mu_{I,s}) + (\sigma-1)\frac{\alpha_{I,s}}{2}(\sigma^2_{T,s} + \sigma^2_{I,s} - 2\sigma_{T,I,s}) + (\sigma-1)(\sigma_{a,T,s} - \sigma_{a,I,s})\right) \tag{35}$$

A higher mean distortion[18] of tangible capital (relative to the intangible distortion) will push firms to employ more intangibles and cause the HK model without intangibles to overestimate the gains of misallocation relative to my model with them. A higher correlation between productivity and tangible distortions will generate the same effect as it will make HK tend to overestimate the productivities of the most productive firms—those that "count" the most for the aggregate reallocation gain. Finally, whichever direction the error goes, a higher intangible share amplifies it, since it accentuates the difference between the two models. As $\alpha_{I,s} \to 0$, my model converges to the HK model and the error disappears.

[17] Using the fact that my method of estimating factor shares implies $\frac{\text{MRPK}_{T,s}}{\text{MRPK}_{I,s}} = \frac{r_T}{r_I}$, which greatly simplifies the lognormal exercise that follows.

[18] Note that the relative distortion is determined not just by log distortion means, but also by their variances and covariances.

4 Quantitative Results

Using expressions provided by the model in the previous section, I compute the TFP/output gain from equalizing distortions across firms. This measures the drag of misallocation on the economy.

4.1 Aggregate Misallocation

Reallocation gain: HK+I vs HK. I start with the aggregate results, based on equation (32). I plot Gain minus 1 in Figure 5 for both the HK+I model (with intangibles) and the original HK (without intangibles). According to both models, misallocation has increased considerably between 1987 and 2017: from 8.7% of TFP to 14.2% according to HK+I and from 9.0% to 21.9% according to HK. The fact that misallocation in the US has increased has been observed by Bils et al. (2018) using the Census of Manufacturers as well[19]. Notice, however, that the measure of misallocation produced by the HK model is consistently higher than that of HK+I. Thus, I come to the first important result: ignoring intangible capital leads us to overestimate misallocation. In 2017, HK exaggerates misallocation relative to HK+I by 7.7 percentage points of actual (distorted) TFP, or by 54.2% in relative terms.

Not only is HK overestimating misallocation relative to HK+I, but the magnitude of this overestimation has grown over time, which can be clearly seen in Figure 6. The difference between the two was just 0.3 percentage points of TFP in 1987 but has increased to 7.7 p.p. in 2017. Therefore, while the output cost of misallocation has increased by 12.9 p.p. of TFP according to HK, at the same time the error of HK (relative to HK+I) has grown by 7.4 p.p. of TFP, accounting for 57% of the increase. Summing up, my second result is that the extent to which the omission of intangible capital exaggerates misallocation estimates has increased over time.

Focusing on the last two decades of the sample paints an even more striking picture. Reallocation gain according to HK+I increased by just 0.5 p.p. between

[19] They attribute this growth in misallocation to increasingly severe mismeasurement.

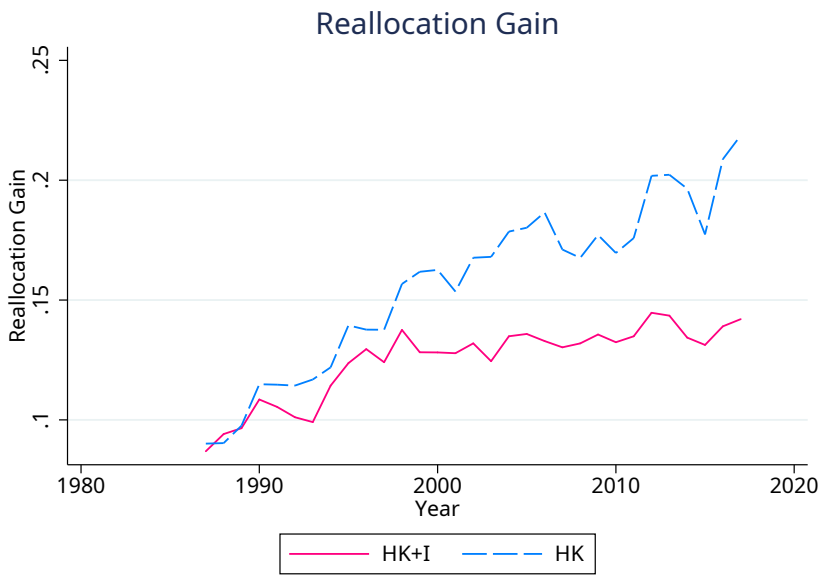


Figure 5: Aggregate Misallocation in HK and HK+I

1997 and 2017, while the one-capital HK model measured an increase of 6.2 p.p. in the same timeframe. Thus virtually all of the worsening of misallocation measured with the standard model in the last two decades disappears once the intangible input is taken into account—the corrected measure of misallocation has hardly budged since 1997.

It must be pointed out that while HK overestimates misallocation in the aggregate for all but one years in the sample, the same is not true for all sectors. As pointed out by equation (35), whether HK over- or underestimates misallocation in a given sector is determined by its intangible share as well as productivity and distortion distributions. Many sectors feature underestimation by HK. Reallocation gain in the largest sectors, however, tends to be overestimated by HK—and it's the large sectors that drive the aggregate result.

Partial reallocations. In addition to quantifying the gain from removing all misallocation of inputs, the model can be used to measure the output gain from equalizing only some of the distortion types (output, tangible, or intangible) while leaving others intact. By comparing such partial reallocation gains to the

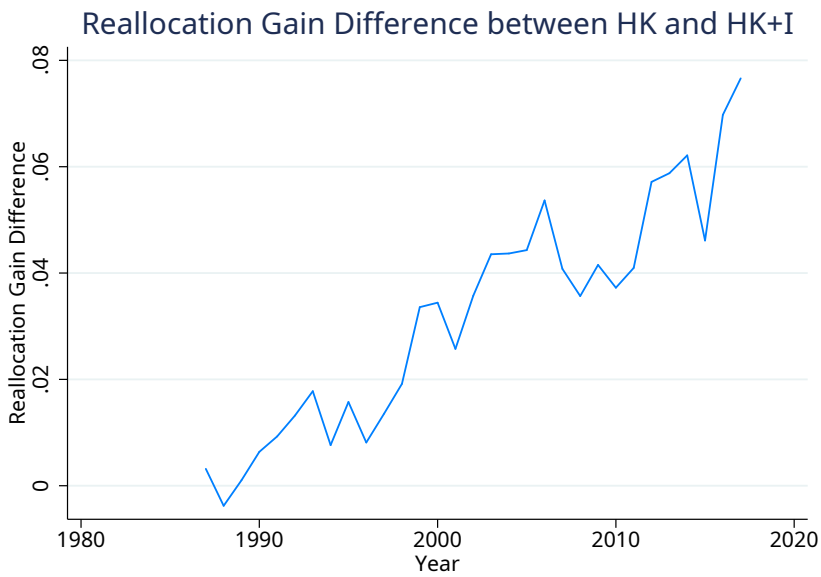


Figure 6: Aggregate Misallocation Overestimation by HK

full reallocation gain, we can evaluate the role of each of the three distortions.

Figure 7 shows how close removing just one distortion type gets us to removing all three. The output distortion has the highest share throughout most of the sample: equalizing $\tau_{Y,s,f}$ across firms in 2017 would increase TFP by almost as much as equalizing all three distortion types would. The role of intangible distortion, however, has grown over time, while the role of the tangible distortion has declined. Note that shares do not sum to one: removing one distortion first would perturb the gains from removing the remaining distortions.

Changing sector importance. How much of the measured misallocation is due to the within-sector, cross-firm misallocation getting stronger, and how much is due to already more distorted sectors simply becoming larger and thus getting a higher weight in the aggregate misallocation measure? I try to address this question by fixing each sector's revenue share θ_s at its 1987 level and evaluating the counterfactual evolution of reallocation gains in the absence of changes in relative sector importance. The outcome of this exercise is shown in Figure 8 with the benchmark misallocation series for comparison. I find that

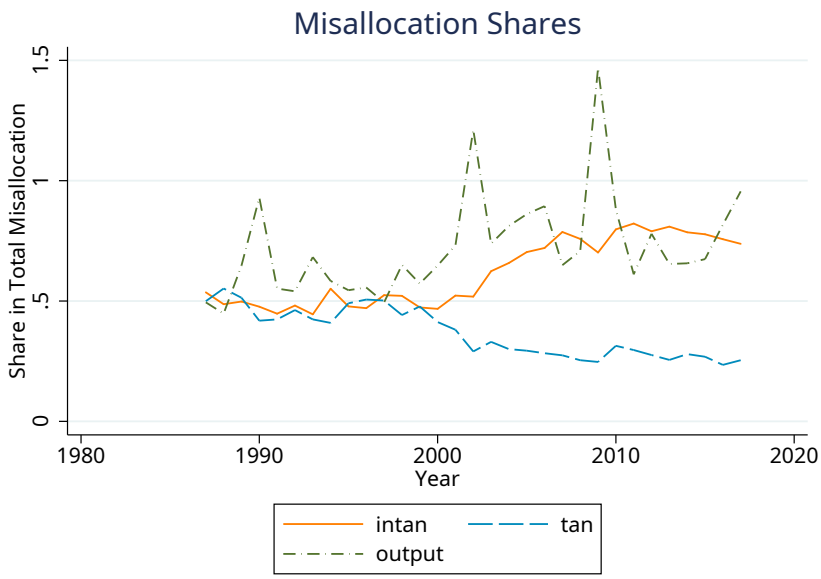


Figure 7: Distortion Shares in Misallocation

the counterfactual economy's misallocation grows from 8.7% to 13.7% of TFP (vs 8.7% $\rightarrow$ 14.2% with changing revenue shares), meaning that the difference is 0.5 percentage points of TFP, or one tenth of the increase. So, a small part of the rise in misallocation over the last three decades is due to sectors with higher misallocation simply getting bigger, with the rest being due to increasing within-sector misallocation.

4.2 Misallocation and Intangible Intensity

I have shown that ignoring intangible assets can cause overestimation of the efficiency costs of misallocation. Now I want to ask a different question: do sectors that employ more intangible capital face more or less severe misallocation than sectors that rely primarily on tangibles?

To answer this, I split all sectors into two groups based on each sector's intangible/tangible capital ratio in 2017. Sectors with an intangible/tangible ratio below the median are assigned to the low-intangible group, while those

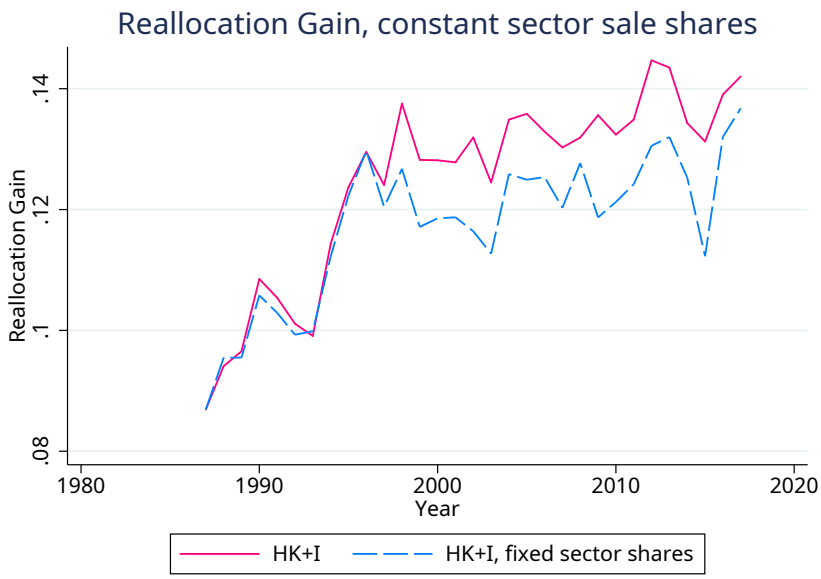


Figure 8: Misallocation with Fixed Sector Sizes

above the median are assigned to the high-intangible group[20]. Then I repeat all reallocation gain computations for each group separately, as if it was a whole economy.

Reallocation gain. Figure 9 presents the overall reallocation gain in each of the two groups. First of all, intangible-intensive sectors have considerably higher misallocation: it is worth 16.3% of TFP in 2017, compared to just 9.3% in tangible-intensive sectors. But not only do high-intangible sectors have a higher *level* of misallocation, they have also experienced somewhat higher *growth* in the cost of misallocation (in both absolute and relative terms): 4.7 percentage points of TFP compared to 3.5 p.p.

A natural explanation would be that this stark difference is driven by high dispersion of measured intangible distortions (caused by high measurement error in my imputation of intangible capital). However, this is not the case: the pattern is still there if I repeat the exercise using the original HK method, which can be seen

[20]The two groups are quite evenly matched in their shares of total sales in 1987 with a 51–49 split between the high- and low-intangible groups respectively, although the high-intangible group's share grows to 71% of the total revenue by 2017: see Figure A.1 in Appendix B.

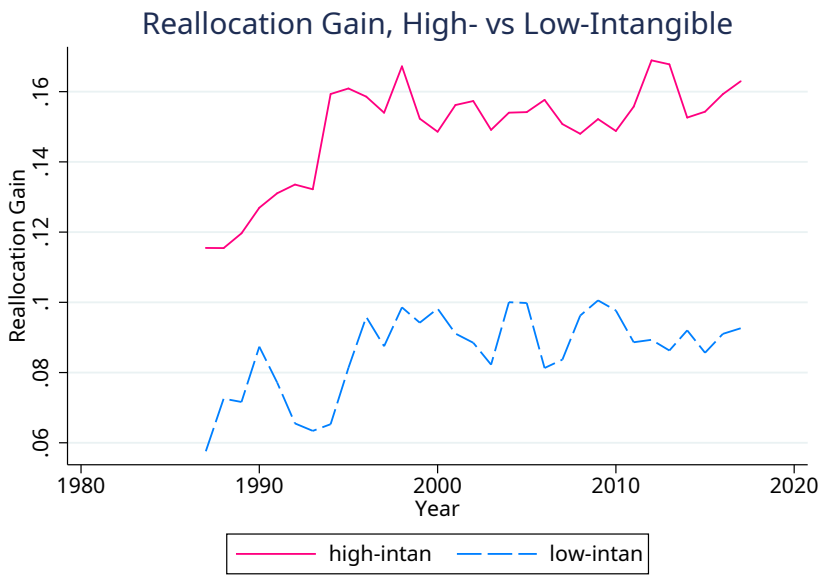


Figure 9: Aggregate Misallocation by Intangible Intensity

in Figure A.2. With the HK method, I use my estimates of firm-level intangibles only to split the sectors into two groups, but not for any misallocation-related estimates.

Revenue productivity dispersions. The difference in allocative efficiency between tangible- and intangible-intensive sectors can also be seen in a less model-dependent way. In Figure 10, I plot the standard deviation of log tfpr (with three inputs[21]) demeaned within sector-year—a measure of within-sector across-firm dispersion often used as a quick and simple way of diagnosing misallocation. The figure indicates a similar pattern: dispersion has been considerably higher in the intangible-intensive sectors throughout the entire sample period[22]. Moreover, while aggregate dispersion in revenue productivities has increased over this period, all of this growth is due to the increasing dispersion in

[21] $\text{tfpr}_{s,f} = (p_{s,f} y_{s,f}) \,/\, \left(k_{T,s,f}^{\alpha_{T,s}} k_{I,s,f}^{\alpha_{I,s}} v_{s,f}^{1-\alpha_{T,s}-\alpha_{I,s}} \right)$

[22] Just as for the HK+I misallocation measure, this result is not driven by my estimates of intangible capital. The dispersion of tfpr that includes only the two standard inputs, $\text{tfpr}_{s,f} = (p_{s,f} y_{s,f}) \,/\, \left(k_{T,s,f}^{\alpha_{T,s}+\alpha_{I,s}} v_{s,f}^{1-\alpha_{T,s}-\alpha_{I,s}} \right)$, produces the same pattern, as shown in Figure A.3.

the high-intangible part of the economy. This result has already been shown by Caggese and Pérez-Orive (2016).

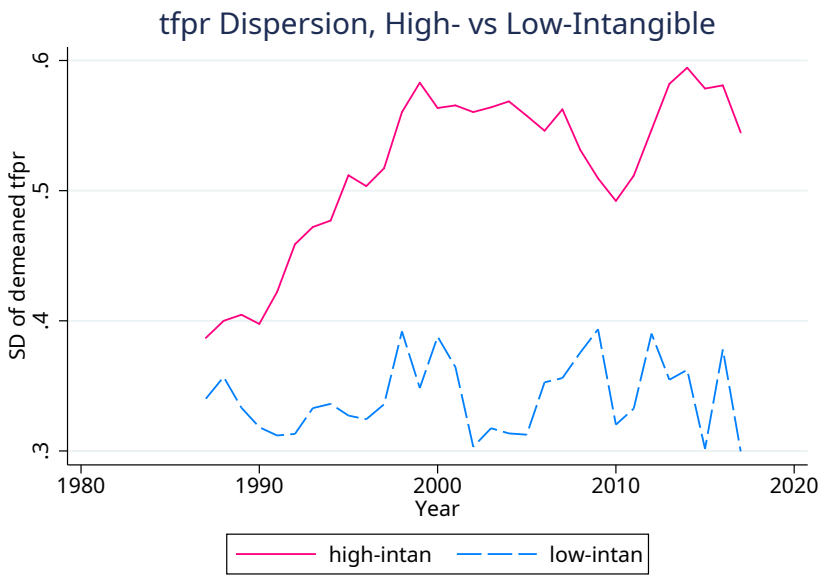


Figure 10: Dispersion of tfpr by Intangible Intensity

4.3 Extensions

Longer sample with a single deflator. The limiting factor on the length of the sample used in the analysis presented above is sectoral deflators obtained from the BLS, which are available starting in 1987 only. To verify my results, I repeat the analysis above after using a single economy-wide deflator: producer price index for all commodities from the BLS. When presenting results, I discard the first two decades of Compustat data, 1950-1969, because data coverage varies widely across sectors and years in that period. This method yields similar estimates. With a long sample and a single deflator, the TFP cost of misallocation increased from 6.4% to 13.4% according to the HK+I model and from 8.9% to 20.1% according to the HK model. The evolution of misallocation is displayed in Figure A.4 in Appendix B. Just as in the benchmark, not only is HK overestimating misallocation relative to HK+I, but the magnitude of the

error is growing over time, from 2.5 percentage points in 1970 to 6.7 points in 2017.

The extension confirms that a small part of the observed increase in aggregate misallocation is due to the more distorted sectors getting bigger, shown in Figure A.5. Computing counterfactual misallocation while fixing sectoral shares at their 1970 levels produces an increase that is 0.6 percentage points smaller.

Figure A.6 illustrates that in this extension, misallocation in the intangible-intensive sector is also considerably higher and has increased more over the years.

Alternative elasticities of substitution. In the main analysis above I assume the elasticity of substitution between varieties (σ) to be 3. Here I check the sensitivity of the results to the choice of this parameter by repeating the reallocation gain exercise for $\sigma = 2$ and $\sigma = 4$. The results are presented in Table 2. Reallocation gains do change with σ, but the result that the standard HK model overestimates misallocation relative to the HK+I model in 2017, and that the degree of this overestimation has growh significantly over time, are present for all three parameter values. Moreover, all three parameterizations give similar values for the portion of the increase in misallocation measured by the HK model that is attributable to its growing error relative to the HK+I model.

Table 2: Reallocation gain results for alternative elasticities of substitution

σ	2		**3**		4	
	1987	2017	**1987**	**2017**	1987	2017
HK+I Gain	7.2%	11.3%	**8.7%**	**14.2%**	11.2%	18.8%
HK Gain	6.7%	14.6%	**9.0%**	**21.9%**	12.4%	30.3%
%Δ	-7.5%	29.2%	**3.4%**	**54.2%**	10.7%	61.2%
HK↑ due to overestim.	48.1%		**57.4%**		57.5%	

Note. The first two rows show the reallocation gain in % of TFP for the HK+I and the HK models respectively. The third row shows the % difference between the two. The last row shows the portion of the increase in misallocation measured by the HK model that is due to its overestimation relative to HK+I.

Intangible-intensive sectors have significantly higher misallocation than tangible intensive sectors for these alternative choices of σ as well. While intangible-intensive sectors have 75% more severe misallocation in the $\sigma = 3$ benchmark, their misallocation is 67% higher with $\sigma = 2$, and 96% higher with $\sigma = 4$.

5 Potential Mechanisms

In the previous section, I showed that sectors that use more intangible capital face more severe misallocation. In this section, I discuss some of the potential mechanisms that could explain this fact.

First of all, which sectors are in which category? Table 3 lists the five most intangible-intensive sectors and the five most tangible-intensive sectors. Motion picture and sound recording industries is the most intangible-intensive sector, while forestry and fishing is the most tangible-intensive one.

Table 3: The most and the least intangible-intensive sectors

$\frac{K_I}{K_T}$ Rank	Sector	$\frac{K_I}{K_T}$
1	Motion picture and sound recording	4.85
2	Computer systems design	4.81
3	Publishing industries	4.47
4	Administrative and support services	4.37
5	Ambulatory health care services	4.13
⋮		
48	Water transportation	.09
49	Petroleum and coal products	.08
50	Oil and gas extraction	.05
51	Utilities	.05
52	Forestry, fishing, and related	.02

Note. Sectors follow the BLS classification. They are ranked by their 2017 ratio of intangible to tangible capital, the same method that is used to construct the high- and low-intangible groups.

It is useful to explore several dimensions in which the tangible-intensive and the intangible-intensive sectors differ. Some of the statistics describing these dimensions are summarized in Table 4. They are discussed separately in the

Table 4: Low- vs high-intangible sectors in 2017

	low-intan	high-intan	Δ
K_T factor share	0.230	0.083	
K_I factor share	0.124	0.260	
V factor share	0.646	0.657	
SD of demeaned log sales	1.95	2.53	0.58
			(0.09)
Median firm growth rate SD	0.17	0.24	0.07
			(0.01)
Median leverage ratio	0.32	0.20	−0.12
			(0.01)
Sales-weighted markup	1.048	1.178	0.13
			(0.06)

Note. Bootstrapped standard errors of the differences in parentheses.

following paragraphs together with the mechanisms that could be generating them.

Winner-take-most. One potential mechanism driving the observed misallocation is that intangible-intensive sectors are winner-take-most environments. Narratively, some of the most intangible-intensive sectors in Table 3 do seem to produce outputs that are to some extent non-rival and imply high returns to scale.

The high-intangible group of sectors also has a higher dispersion of firm sizes. In the intangible-intensive group of sectors, demeaned (within sector-year) firm log sales in 2017 have a standard deviation that is 30% higher than in the tangible-intensive group. Of course, this fact could be driven by many other stories, and more empirical work is needed to find evidence for or against this mechanism.

Uncertainty. Firms in intangible-intensive sectors have more volatile sales growth. I take the standard deviation of the year-to-year sales growth rate within each firm. In high-intangible sectors, the median firm-level growth rate volatility

is 40 percent higher than in the low-intangible sectors. This relationship can also be seen at the less disaggregated sectoral level, shown in Figure 11.

This result suggests that firms in intangible-intensive sectors may be facing more uncertainty for one reason or another. It can be a feature of the markets they operate in (more uncertain demand), or it can be a feature of their production. Intangible investment may have highly volatile productivity, for example. A given R&D investment might yield a breakthrough design one year and nothing of value the next. This uncertainty will lead to measured misallocation as two otherwise equal firms may land with different quantities of the intangible input.

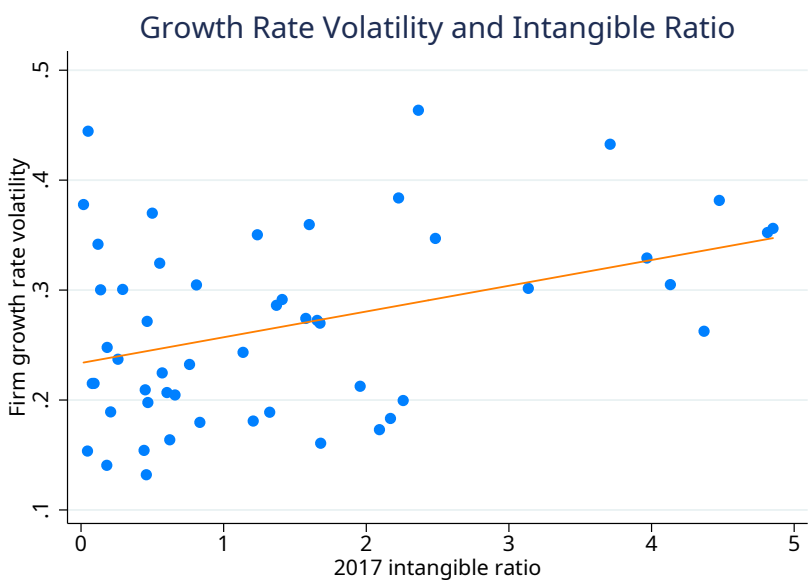


Figure 11: Firm growth volatility and intangible intensity by sector

Financial frictions. Firms in intangible-intensive sectors borrow less. Defining firms' leverage as $\frac{\text{total debt}}{\text{total assets}}$, its median is 60% higher in the low-intangible group of sectors. Defining leverage as $\frac{\text{total debt}}{k_T+k_I}$ instead, the median is 100% higher in sectors that use more tangibles.

Financial frictions are often used in the intangibles literature to explain various differences between low- and high-intangible firms and sectors. They are

frequently modeled with collateral constraints, relying on the notion that it is easier to pledge factories and machines (tangible capital) rather than ideas and organizational innovations (intangible capital) as collateral[23].

In the context of misallocation, however, and in particular in application to the data I am using, I believe that the collateral constraints mechanism is not the most fruitful approach. Firstly, it is hard to imagine that considerations of collateral are important for the financing decisions of Compustat firms—publicly traded and, overwhelmingly, large. Secondly, the existence of collateral constraints on its own can generate capital misallocation between tangible- and intangible-intensive sectors, but additional features are needed to explain greater misallocation *within* high-intangible sectors (some intangible-intensive firms have to be more financially constrained than other intangible-intensive firms).

A more appealing financial friction is the uncertainty of the outcome of intangible investment coupled with private information on the potential of the investment opportunity, in the spirit of Leland and Pyle (1977). In their model, entrepreneurs possess private information on the expected outcome of the project and signal this information to investors by choosing how much of the project to finance internally. I believe that, if explored, this mechanism can prove to be empirically relevant in the financing of intangibles.

Markups. Intangible-intensive sectors have a higher aggregate markup. I measure markup as $\mu_{s,f,t} = (1-\alpha_{T,s}-\alpha_{I,s})\frac{p_{s,f,t}y_{s,f,t}}{w_{s,f,t}v_{s,f,t}}$, where $w_{s,f,t}v_{s,f,t}$ is measured with cost of goods sold. The sales-weighted average of firms' markups is 1.18 in the high-intangible sectors in 2017, compared to just 1.05 in the low-intangible sectors. There is little difference in median markups, however: the median markup is 1.01 in the high-intangible sectors and 1.00 in the low-intangible ones. This suggests that the higher aggregate markup is driven not by all firms in the intangible-intensive sectors enjoying higher market power, but rather by a stronger relationship between size and markups in those sectors.

[23]For example, see Pérez-Orive (2016), Falato et al. (2013), or Caggese and Pérez-Orive (2016).

6 Variable Markup Model

In this section, I explore one mechanism that could potentially be driving the higher misallocation in high-intangible sectors: markup heterogeneity caused by the uncertain productivity of intangible investment. While in the real world, tangible investment projects are likely to face uncertain returns as well, the outcome of R&D endeavors or of attempts to reform the organizational structure is arguably much more variable. Furthermore, intangible investment projects require a long-term commitment of highly skilled labor, with parts of the resulting asset being embedded in the human capital of the employees. Based on this, I model intangible capital as less readily adjustable than its conventional counterpart[24]. The model is essentially a partial equilibrium version of the model from Edmond et al. (2018), but with intangibles added.

6.1 Model

Overview. I model a small sector whose firms take aggregate variables as exogenous and constant—and thus not affected by the sectoral equilibrium quantities. A continuum of firms enter at $t = 0$ and produce in all the following $t \geq 1$. Upon entering, firms make an intangible investment decision x_I, after which each firm draws the productivity $e \sim G(e)$ of its intangible investment, endowing the firm with $k_I = e \cdot x_I$ units of intangible capital. Following the draw, each firm chooses the amount of tangible investment x_T, which deterministically results in $k_T = x_T$ units of conventional capital. Both capital quantities are fixed for all the subsequent periods to reflect adjustment costs[25]. Once the capital quantities have been determined, firms produce in all the following periods, choosing their variable input and quantity. The market is monopolistically competitive, but

[24]See Brown et al. (2009), Hall and Lerner (2010), and Brown and Petersen (2011) for more discussion of the high adjustment costs of R&D (which I believe applies to other types of intangibles as well). For a view that the relative smoothness of R&D expenditures may reflect not higher adjustment costs, but rather different factors affecting R&D investment and conventional investment, see Lach and Schankerman (1989).

[25]Intangible investment is picked first to reflect the greater severity of intangible adjustment costs.

the sectoral good aggregator is such that each firm faces an elasticity of demand that is increasing in the firm's relative quantity produced. Firms thus face variable markups that increase in size. The dispersion of markups creates measured misallocation of inputs across firms.

Timeline. All the action happens in the initial period of the model. All the consecutive periods are identical to each other—there is no uncertainty and no dynamic decisions in $t \geq 1$. The following is the timeline of each firm's life:

$\underline{t = 0:}$

- sub-period 0: enter
- sub-period 1: choose intangible investment x_I
- sub-period 2: draw intangible investment productivity e to obtain $k_I = ex_I$
- sub-period 3: choose tangible investment $x_T = k_T$

$\underline{t \geq 1:}$ produce (choose output y_t and variable input v_t each period)

Market. The sectoral composite good is produced competitively from varieties $y_t(\omega)$ using the Kimball aggregator:

$$\int_0^{F_t} \Upsilon\left(\frac{y_t(\omega)}{Y_t}\right) d\omega = 1 \tag{36}$$

I use the functional form for Υ from Klenow and Willis (2016):

$$\Upsilon(q) = 1 + (\tilde{\sigma} - 1)\exp\left(\frac{1}{\varepsilon}\right)\varepsilon^{\frac{\tilde{\sigma}}{\varepsilon}-1}\left(\Gamma\left(\frac{\tilde{\sigma}}{\varepsilon}, \frac{1}{\varepsilon}\right) - \Gamma\left(\frac{\tilde{\sigma}}{\varepsilon}, \frac{q^{\frac{\varepsilon}{\tilde{\sigma}}}}{\varepsilon}\right)\right) \tag{37}$$

where $q = \frac{y}{Y}$ and $\Gamma(s, x) = \int_x^\infty t^{s-1}e^{-t}dt$.
The intermediate demand for variety ω is then

$$p_t(\omega) = \Upsilon'\left(\frac{y_t(\omega)}{Y_t}\right) D_t \tag{38}$$

where

$$D_t = \left(\int_0^{F_t} \Upsilon' \left(\frac{y_t(\omega)}{Y_t} \right) \frac{y_t(\omega)}{Y_t} d\omega \right)^{-1} \tag{39}$$

and $\Upsilon'(q) = \frac{\tilde{\sigma}-1}{\tilde{\sigma}} \exp\left(\frac{1-q^{\frac{\varepsilon}{\tilde{\sigma}}}}{\varepsilon}\right)$.

Firms. Each firm produces a variety y from tangible capital k_T, intangible capital k_I, and variable input v using Cobb-Douglas technology. Since k_T and k_I are fixed after the initial period, $z = k_T^{\alpha_T} k_I^{\alpha_I}$ is the productivity of the variable input that the firm is stuck with. The distribution $G(e)$ and optimal policy functions $k_I(e)$ and $k_T(k_I)$ generate the distribution $H(z)$.

$$y_t = \underbrace{k_T^{\alpha_T} k_I^{\alpha_I}}_{\equiv z} v_t^{1-\alpha_T-\alpha_I} \tag{40}$$

Static production problem in $t \geq 1$. The firm produces in each period starting with 1, choosing output and variable input to maximize profit:

$$\pi(z) = \max_{y_t, v_t} p(z)y(z) - w_t v(z) = \max_{y_t} \frac{\tilde{\sigma}-1}{\tilde{\sigma}} \exp\left(\frac{1 - \left(\frac{y_t}{Y_t}\right)^{\frac{\varepsilon}{\tilde{\sigma}}}}{\varepsilon} \right) y_t D_t - w_t \left(\frac{y_t}{z}\right)^{\frac{1}{1-\alpha_t-\alpha_I}} \tag{41}$$

The first-order condition is

$$p(z) = \underbrace{\frac{\tilde{\sigma}\left(\frac{y(z)}{Y}\right)^{-\frac{\varepsilon}{\tilde{\sigma}}}}{\tilde{\sigma}\left(\frac{y(z)}{Y}\right)^{-\frac{\varepsilon}{\tilde{\sigma}}} - 1}}_{\mu\left(\frac{y(z)}{Y}\right)} \underbrace{\frac{1}{1-\alpha_T-\alpha_I} w \left(\frac{y(z)}{z}\right)^{\frac{1}{1-\alpha_T-\alpha_I}} \frac{1}{y(z)}}_{MC(z)} \tag{42}$$

<u>Key mechanism.</u> Price is markup over marginal cost, but unlike the CES case, markup $\mu\left(\frac{y(z)}{Y}\right)$ is increasing in the firm's relative output $\frac{y(z)}{Y}$: larger firms charge higher markup. This is driven by the non-CES sectoral aggregator defined in (36) and (37). It generates size-dependent markups similarly to oligopolistic

competition models like Atkeson and Burstein (2008), but does it in a much more tractable way.

Define $A = \frac{w}{D} Y^{\frac{1}{1-\alpha_T-\alpha_I}-1}$, which summarizes all the aggregate information the firm needs to know, and rewrite the first-order condition:

$$\Upsilon'(q(z,A))q(z,A) = \mu(q(z,A))\frac{A}{1-\alpha_T-\alpha_I}\left(\frac{q(z,A)}{z}\right)^{\frac{1}{1-\alpha_T-\alpha_I}} \tag{43}$$

$t = 0$, sub-period 3. Given the firm's draw of intangible capital in sub-period 2, the firm chooses tangible capital to maximize its future flow of profits less the cost of capital. Firm owners discount future profits at β. Moreover, firms face an exogenous death probability d.

$$\max_{k_T} -k_T(k_I) + \beta \sum_{t=1}^{\infty} (\beta(1-d))^{t-1} \pi_t \left(k_T^{\alpha_T}(k_I) k_I^{\alpha_I}\right) \tag{44}$$

Solving and combining with the static FOC, arrive at the tangible investment first-order condition:

$$k_T(k_I) = \alpha_T \beta \sum_{t=1}^{\infty} (\beta(1-d))^{t-1} \frac{p_t(k_I) y_t(k_I)}{\mu_t(k_I)} \tag{45}$$

where $p_t(k_I), y_t(k_I), \mu_t(k_I)$ stand for $p_t(k_T^{\alpha_T}(k_I)k_I^{\alpha_I})$, etc.

$t = 0$, sub-period 2. The firm draws the productivity of the intangible capital production function: $e \sim G(e)$. Investing x_I yields $k_I = ex_I$.

$t = 0$, sub-period 1. Firms choose intangible investment to maximize expected profit less investments into two capitals. Since productivities have not been drawn yet, all firms are identical at this point and will pick the same x_I.

$$\max_{x_I} -x_I^{\phi} + \int \left(\beta \sum_{t=1}^{\infty} (\beta(1-d))^{t-1} \pi_t \left(k_T^{\alpha_T}(ex_I)(ex_I)^{\alpha_I}\right) - k_T(ex_I)\right) dG(e) \tag{46}$$

where $\phi > 1$. Note that the intangible investment cost function is convex. This is needed to close this partial equilibrium model, otherwise the scale of the solution is not fixed.

The solution algorithm is described in Appendix A.

Aggregate markup. Aggregate markup is an important quantity for calibration. It is the sales-weighted average of firms' markups:

$$M = \int \mu(z) \frac{p(z)y(z)}{\int p(z')y(z')dH(z')} dH(z) \tag{47}$$

6.2 Calibration

Targeted moments. I assume e to be lognormally distributed, with $SD[\log(e)] = \iota$. Factor shares α_T and α_I come directly from the data: they are the averages of the respective factor shares of industries included in calibration, weighted by each industry's sales share.

Three parameters need to be calibrated: the average elasticity of demand $\tilde{\sigma}$, the superelasticity ε, and the standard deviation of log productivity ι. I describe the moments I use to pin these below, but a more thorough discussion can be found in Edmond et al. (2018).

The average elasticity of demand $\tilde{\sigma}$ is picked to match the aggregate markup M. The firm's static FOC implies that $\mu(z) = (1 - \alpha_T - \alpha_I)\frac{p(z)y(z)}{wv(z)}$. I measure $wv(z)$ as cost of goods sold (COGS).

The superelasticity ε is picked to match the relationship between the relative variable input (i.e. demeaned within industry-year) and relative sales (similarly demeaned). I group firms in the data into deciles by their relative sales and take the average log relative sales within each decile. The model predicts certain log relative variable input for each log relative sales number. I then compare the predicted variable input with the observed log average input in each decile. Superelasticity is chosen to minimize the distance between the vector of predicted variable input quantities and the vector of observed average variable input quan-

tities.

The dispersion of log productivity of intangible investment ι is chosen to match the distribution of relative sales (demeaned within industry-year). I measure the log relative sales at various percentiles of the distribution and pick ι that best approximates the resulting vector in the model.

Calibration strategy. My model is partial equilibrium, and so I solve it independently for the low- and high-intangible sectors. The intangible intensity of each sector is determined by its factor shares, which I take from the low- and high-intangible sector groups constructed in Section 4.2. These and all other moments are calculated for 2017.

I compute the moments described above for each of the two sectors separately. Then I calibrate the tangible-intensive sector to match its data moments. Afterward, instead of calibrating the intangible-intensive sector to its moments, I plug the parameters calibrated for the low-intangible sector into the high-intangible one and check how well the model reproduces the moments of the high-intangible sector, which are not targeted. The objective of this exercise is to see how much of the difference between the two sectors the model can explain when the only parameters that are different between the two sectors are the output elasticities of the two capital inputs. Since my objective is to study the role of intangible assets, rather than the role of markups, in generating differences between the two sectors, I believe that this approach is more reasonable than calibrating *both* sectors to their respective moments.

Chosen and calibrated parameters are listed in Table 5. The values of the three key parameters that best match the data moments are: average elasticity $\tilde{\sigma} = 29$, superelasticity $\varepsilon = 6$, and intangible investment productivity dispersion $\iota = 2.25$.

6.3 Results

Matching data moments. The model is able to match calibrated moments reasonably well. Table 6 shows that for the low-intangible sector (to which all parameters were calibrated), aggregate markup is matched perfectly,

Table 5: Parameter values

Parameter	Symbol	Value	Rationale
discount rate	β	0.96	standard
death rate	d	0.10	standard
intan inv convexity	ϕ	1.1	ad hoc
Low-Intan Sector			
tangible share	$\alpha_{T,l}$	0.230	compensation share
intangible share	$\alpha_{I,l}$	0.124	compensation share
average elasticity	$\tilde{\sigma}$	29	aggregate markup
superelasticity	ε	6	sales-COGS relationship
productivity dispersion	ι	2.25	sales distribution
High-Intan Sector			
tangible share	$\alpha_{T,h}$	0.083	compensation share
intangible share	$\alpha_{I,h}$	0.260	compensation share

the distribution of relative sales is matched tolerably well, but the relationship between relative sales and relative variable input is reproduced imperfectly.

The ability of the model to match data moments from the non-targeted intangible-intensive sector is not good. The model does produce the qualitative differences between the two sectors: the aggregate markup, the dispersions of relative sales, and the predicted relative COGS are all higher in the intangible-intensive sector both in the data and in the model. But quantitatively the model is unable to match the difference between the two sectors in these moments. It falls particularly short in explaining the difference in aggregate markups.

A much better fit could be obtained if the high-intangible sector was calibrated separately to its own moments (in my analysis, the intangible-intensive sector uses parameters from the low-intangible sector, and the only difference between them is factor shares). It is natural to suppose that in reality, high-intangible industries produce goods with different substitutability and face a different dispersion of intangible investment productivity compared to other industries. But allowing all parameters to vary between the two sectors would obscure the role of intangible assets per se, because it is just as natural to be-

lieve that the share of intangible capital in production is far from the only thing that differs between the two groups of industries.

Table 6: Performance of the model in matching data moments

	low-intan (targeted)		high-intan (non-targeted)	
	data	model	data	model
Aggr. Markup				
M	1.048	1.048	1.178	1.053
Relative COGS				
decile				
1st	-4.20	-2.47	-5.17	-3.70
2nd	-3.01	-1.64	-4.14	-2.77
3rd	-2.06	-1.07	-3.52	-2.16
4th	-1.62	-0.77	-2.79	-1.67
5th	-1.22	-0.50	-2.32	-1.24
6th	-0.85	-0.21	-1.57	-0.78
7th	-0.34	0.11	-1.20	-0.39
8th	0.06	0.35	-0.47	0.05
9th	0.75	0.79	0.27	0.54
10th	1.64	1.31	2.00	1.66
Relative Sales				
percentile				
20th	-3.22	-2.83	-4.23	-3.47
40th	-1.72	-1.73	-2.70	-2.25
60th	-0.77	-0.82	-1.54	-1.22
80th	0.24	0.18	-0.23	-0.06
90th	0.98	0.89	0.74	0.79
95th	1.43	1.46	1.49	1.47
99th	2.28	2.46	2.77	2.69

Measured misallocation. Figure 12 illustrates the key mechanism of the model: firms that drew a better productivity of their intangible investment are able to charge higher markups, and the relative market power enjoyed by the most productive firms is greater in the intangible-intensive sector. As productivity increases, costs fall, but prices are not reduced by as much, raising the overall markup. This also produces a revenue productivity that is increasing in intangible investment productivity, displayed in Figure 13. Moreover, the gap

between the tfpr of productive firms on one hand and unproductive ones on the other is starker in the intangible-intensive sector. The HK framework would interpret this tfpr dispersion as evidence of misallocation.

The HK framework relies on the measurement of marginal revenue products which let us impute the underlying distortions. As described in Section 3.2, mrpk_T, for instance, is measured in the data as $\frac{\sigma-1}{\sigma}(1-\alpha_{T,s}-\alpha_{I,s})\frac{y_{s,f}p_{s,f}}{k_{T,s,f}}$. In this model, the true marginal product is different, hence I refer to the preceding expression as "naive mrpk_T". All three naive marginal products (wrt k_T, k_I, and v) follow a pattern similar to tfpr and are not shown here. As productivity rises, so do the naive marginal products: very productive firms act monopolistically, restrict output, and employ "too few" inputs.

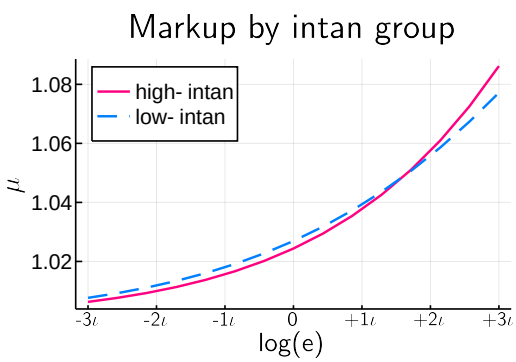


Figure 12: Markup in the Model

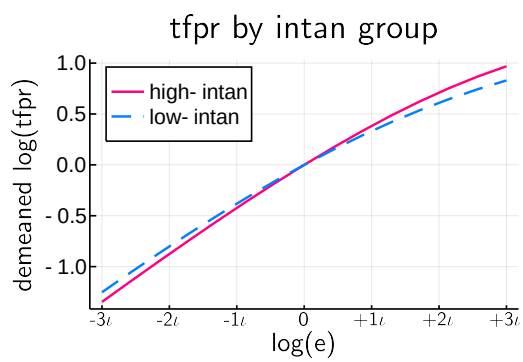


Figure 13: tfpr in the Model

The main test of this model is whether it is able to reproduce the observed misallocation, especially the difference in misallocation severity between low- and high-intangible sectors. To answer this question, I can use the HK+I expressions from Section 3.3 to figure out how much misallocation the HK+I model would measure if this variable markup model was an accurate description of reality. The results are summarized in Table 7. I find that in the low-intangible sector, the measured reallocation gain from equalizing distortions ($\textsc{Gain}_s - 1$) is 17.0% of actual TFP (compare to 9.3% in the data), while the reallocation gain in the high-intangible sector is 19.7% (16.3% in the data). Using the sales share of each sector in the data and Equation 32 to compute the aggregate reallocation gain, the overall measured cost of misallocation is 18.9% of TFP (compare to

14.2% in the data). So, the model is able to produce measured misallocation as well as a higher level of misallocation in the intangible-intensive sector. It does, however, overestimate the reallocation gain in the less misallocated sector and underestimate the gain in the more misallocated sector, ultimately explaining just a third of the gap between the two.

Table 7: Measured misallocation in the model

	low-intan		high-intan	
	data	model	data	model
HK+I TFP Gain	9.3%	17.0%	16.3%	19.7%
tfpr Dispersion	0.300	0.353	0.545	0.397

The tfpr dispersion in the model is 0.353 in the low-intangible sector and 0.397 in the high-intangible sector. In the data, these dispersions are 0.300 and 0.545 respectively. Again, the model underestimates the difference between the two, generating just over a sixth of the empirical gap.

Keep in mind that this measured misallocation is not the *true* misallocation in this variable markup model. Since markup variation arises as a consequence not of market imperfections, but of aggregator specification, a social planner would not achieve a much higher level of TFP[26].

6.4 Model Discussion

Model qualities. The model is successful in approximately reproducing the observed misallocation and some of the difference in misallocation between the low- and high-intangible sectors. Moreover, it is qualitatively consistent with some of the empirical patterns described in Section 5. In the model as in the data, the intangible-intensive sector has a higher weighted average markup and a more dispersed size distribution. Another empirical pattern consistent with the model is the negative relationship between firm size and intangible/tangible ratio.

[26]For an extensive treatment of true misallocation in this model, see Edmond et al. (2018).

Size & intangibles. Table 8 shows the results of regressing a firm's ratio of intangible to tangible capital on its sales quintile (within the firm's 3-digit sector-year) in the data, with sector-year and firm fixed effects. The empirical relationship between the firm size and the ratio of the two capitals is strongly negative and monotonic: larger firms have relatively less intangible and relatively more tangible capital.

How is this consistent with the model? Note that there is a crucial disconnect between what I refer to as intangible capital in the model and in the data. In the model, intangible capital depends on intangible investment and its realized productivity. In the data, I only observe the former and construct a measure of intangible capital from a series of investment expenditures. So what I measure as k_I/k_T in the data is actually more closely linked to x_I/x_T in the model: investment series are the only thing I can measure, and I am left in the dark about the true quantities of capital that firms have obtained from their investments.

Now, in the model, the negative relationship between sales py and investment ratio x_I/x_T is produced directly. All entering firms pick the same x_I, and those that draw a good productivity e obtain a high k_I (again, *unobserved* in the data). These firms go on to invest a lot into regular capital (high x_T) and sell a lot (high py). As in the empirical result from Table 8, the bigger the firm, the more skewed toward tangibles will be its investment choice.

7 Conclusion

The role of intangible assets in production is rising, and so are the measured costs of misallocation. The main contribution of this paper is to document the effects of the shift toward intangible-intensive production on measured misallocation.

I show that omitting intangibles leads to an overestimation of the level and growth of misallocation, and that the degree of overestimation gets worse over time. This worsening explains most of the observed deterioration of allocative efficiency in the last twenty years. Estimating misallocation separately for the low- and high-intangible groups of sectors, I also show that the allocative efficiency is much poorer in the intangible-intensive half of the economy.

Table 8: Firm size and k_I/k_T ratio (data)

		k_I/k_T
Sales Quintile	1st	0.000
		(.)
	2nd	-2.056***
		(0.083)
	3rd	-3.279***
		(0.102)
	4th	-3.738***
		(0.121)
	5th	-4.350***
		(0.150)
Adjusted R^2		0.671
Sector-Year FE		Yes
Firm FE		Yes
Observations		124,471

Note. Standard errors in parentheses. *** $p < 0.01$

I propose a simple partial equilibrium model with variable markups to explain the contrast between the tangible- and intangible-intensive sectors. In it, variation in the outcomes of intangible investment and size-dependent markups generate higher dispersion of measured marginal products and revenue productivities in sectors that employ less conventional capital. The model is qualitatively consistent with several features of the data and can quantitatively generate a lot of the observed misallocation but underperforms in some important respects.

To be able to improve on its ability to explain the data, the next iteration of the model, in addition to being general equilibrium and allowing for dynamic capital accumulation, should combine three core features of intangible investment that I believe to be crucial in making it behave differently from conventional capital. Firstly, high uncertainty of its productivity: output of a factory is more predictable than that of an R&D project. Secondly, high adjustment costs: most intangibles are produced by long-term employment of high-skilled workers and are embodied in them. And, finally, increasing returns to scale: the design of

a robot is more scalable than the robot itself. Combining these features with a goods market structure that generates variable markups and a financial market structure that generates frictions in a dynamic general equilibrium model may very well prove sufficient to explain the salient features of the data.

A significant portion of figuring out why some countries are able to produce more from the same amount of labor and capital lies in understanding how assets are distributed across firms and why. But capital is increasingly not just machines, but also machine learning algorithms; not just concrete structures, but also organizational structures. Production of goods requires all of this capital. We shouldn't discard half of it.

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Appendix

A Solution Algorithm

0. Preparation.

   Set up grids: for e of size n_e, k_T of size n_T, k_I of size n_I, z of size n_z, and A of size n_A.

   Define the lower bound of the z grid as the smallest possible z with given grids of e, k_T, and x_I (i.e. when e, k_T, x_I are all at their smallest possible values) and the upper bound as the largest possible z.

   Guess a distribution of zs: for each e, guess the resulting z, and set $Pr(z) = Pr(e)$. In particular, need to guess some x_I that everyone picks, and guess a $k_T(e)$ policy for each e: these will give a z for each e. Note that the resulting zs can be between the grid points of the z grid.

1. Solve the static problem.

   Solve the firm's static problem on the (z, A) grid.

2. Solve for aggregates.

   For the given distribution of zs, find A that solves

$$\int \Upsilon(q(z, A))dH(z) = 1$$

   where $q(z, A)$ needs to be interpolated for each existing z.

   Knowing A, compute $q(z) = q(z, A)$. From here, directly obtain $p(z)$, $\mu(z)$, D, and Y.

3. Solve the sub-period 3 problem.

   Solve for the optimal $k_T(k_I)$ for each point on the k_I grid (i.e. on the $e \times x_I$ grid). Note that p, y, and μ need to be interpolated.

4. Solve the sub-period 1 problem.

   Solve for optimal k_I.

5. Iterate.

   Compute the new $z = k_T(e \cdot x_I)^{\alpha_T}(e \cdot x_I)^{\alpha_I}$ for each e. This gives a new $dH(z)$.

   Repeat starting with step 2 up to here until $dH(z)$ converges.

B Additional Figures

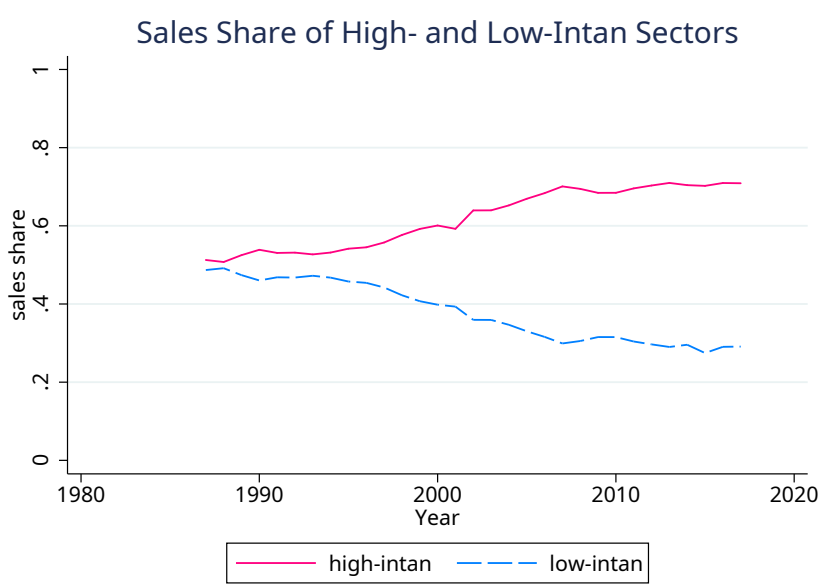

Figure A.1: Sales Shares of Sector Groups

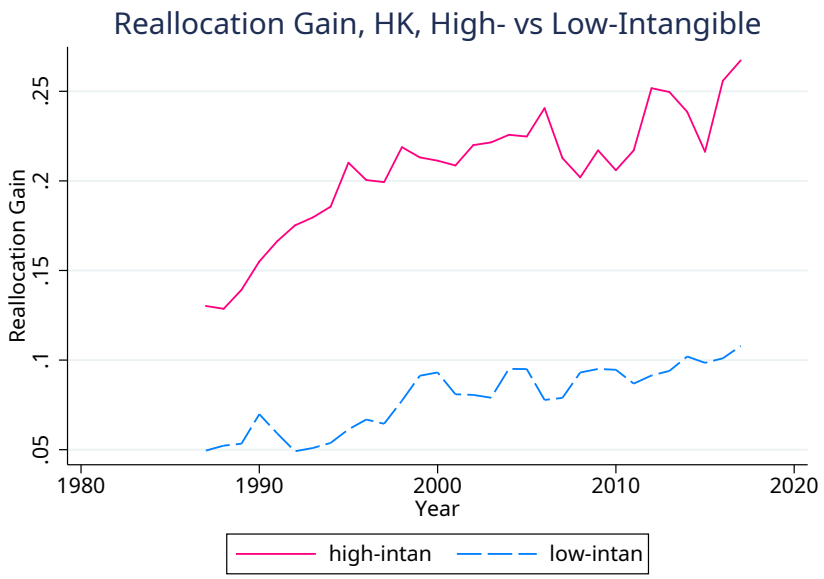

Figure A.2: Aggregate Misallocation by Intangible Intensity, HK

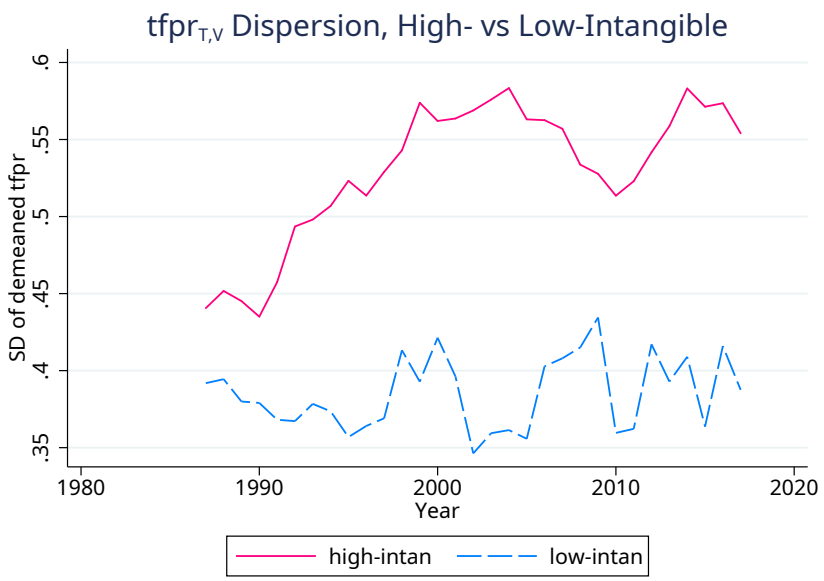

Figure A.3: tfpr Dispersion by Intangible Intensity, Two Inputs

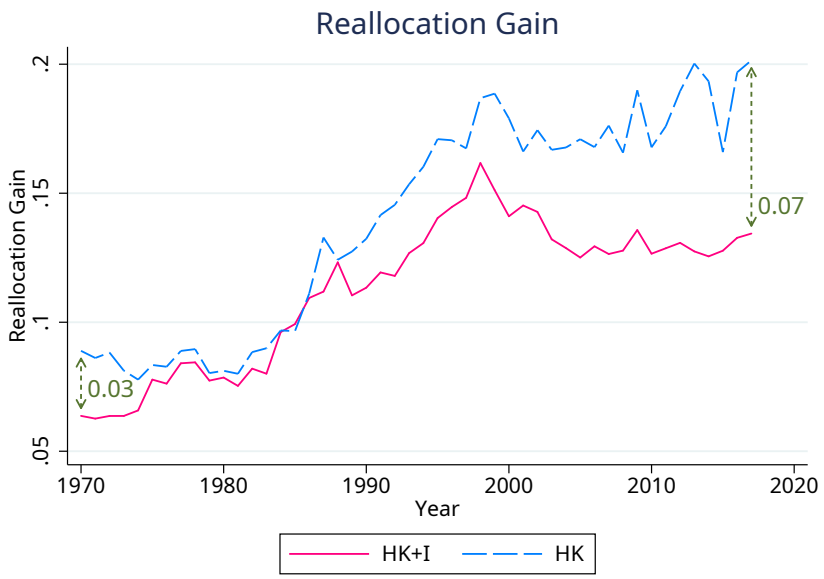

Figure A.4: Aggregate Misallocation, Long Sample Extension

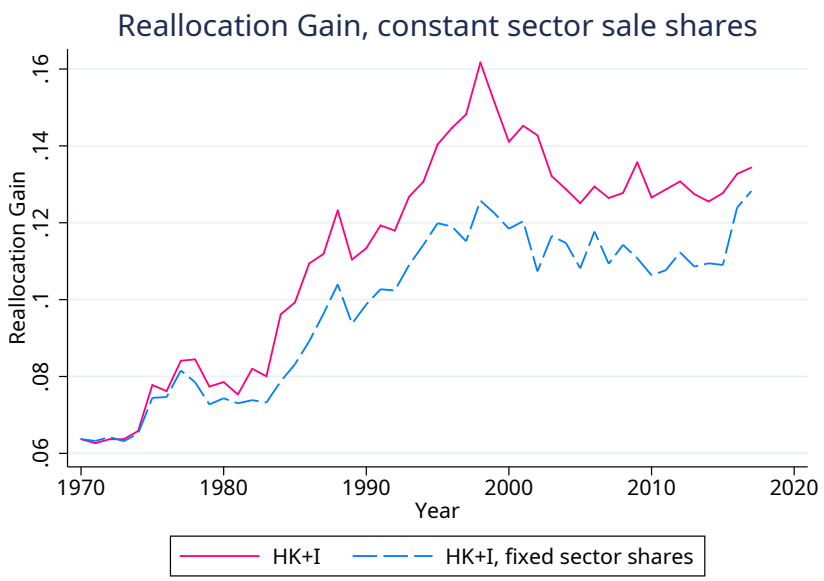

Figure A.5: Misallocation with Fixed Sector Sizes, Long Sample Extension

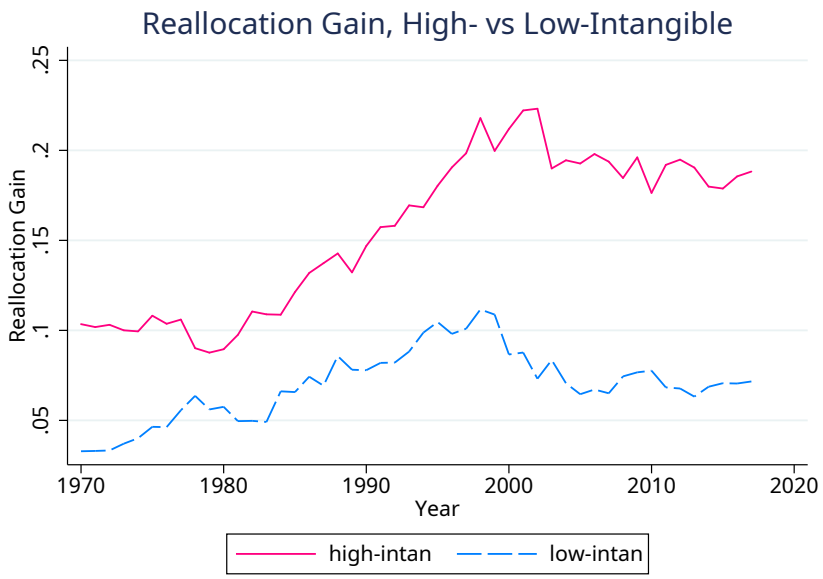

Figure A.6: Aggregate Misallocation by Intangible Intensity, Long Sample Extension